QUANTUM AND CLASSICAL SIGNALS MODELS OF FREQUENCY STANDARDS

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ABSTRACT/KEYWORDS

Quantum model of electromagnetic (EM) signals of frequency standards was suggested, and the properties of this model were established. Transfer from quantum model of EM signal to its classical description was fulfilled.

Keywords: signal, spectrum, quantum model, classical approach, link.

INTRODUCTION

Electromagnetic signals of active microwave and optical frequency standards are quantum systems, and comprehensive in-depth analysis of these signals should be based on quantum physics methods. Alternatively, the results of observation of physical processes and the measurements of their characteristics are expressed in classical forms. The establishing of a link between quantum description of EM signals and the results of their apparatus analysis is important problem of quantum theory measurements. This is a fundamental problem of joining of quantum and classical physics, the developing of unique approach to quantum object and measuring apparatus, interacting with this object. This global problem is yet unsolved [Ref.1].

The research of interaction of EM signals with measuring devices is possible in two aspects. The first of them consists in the introduction of quantum model of EM signal and transfer to its classical representation, whish is adequate to existing description methods of measuring devices operation. The second aspect consists in the developing of quantum theory of measurements, specifically spectral measurements. In this case analyzed signal should be described in quantum physics terms. These reasons require the developing of physical signal theory, based on quantum representations.

This paper is devoted to the solution of problem of quantum and classical physics joining in the context of investigation of EM signals of frequency standards.

QUANTUM MODEL OF ELECTROMAGNETIC SIGNAL AND ITS PROPERTIES

In this report frequency standards signals are considered as free EM fields. In the context of exact, quantum approach EM signal is quantum system, consisting of photons and having limited energy over limited time interval [Refs. 2, 3], or in limited volume. Quantum oscillating process, inducing EM radiation, is the result of action of birth operators at instant of time $t = t_j$ in certain point $\mathbf{r}^0 = (x^0, y^0, z^0)$ of space. The precision of these values is defined by indeterminateness. Born photons have stochastic characteristics: circular frequency $\omega_{\mathbf{k}\alpha j}$; wave vector $\mathbf{k}_{\alpha j} = \mathbf{n}_{\mathbf{k}\alpha j}\hbar\omega_{\mathbf{k}\alpha j} / c_0$ ($\mathbf{n}$ is unit vector, c_0 is velocity); polarization state α or polarization vector $\mathbf{e}$; energy $E = \hbar\omega_{\mathbf{k}\alpha j}$ ($\hbar$ is Planck constant); impulse $\mathbf{p} = \mathbf{k}\hbar$; photon spin orientation (+1, if spin is parallel to its pulse, and –1, if spin is anti- parallel to its pulse).

It is assumed that signals propagate without attenuation along with the fact that photons are steady particles it allow at the forming and propagation of EM signals to take into account only the results of actions of birth operators.

The energy of EM field [Ref 4]

$$E = \sum_{\mathbf{k}\alpha} N_{\mathbf{k}\alpha}\hbar\omega_{\mathbf{k}}, \tag{1}$$

where $N_{\mathbf{k}\alpha}$ is quantum numbers.

Energy (1) is contained in parallelepiped with volume $\Delta v = \Delta s \Delta l_z$ (Δl_z is the height of parallelepiped, $\Delta s = \Delta l_x \cdot \Delta l_y$ is its base). If Δv is unit volume, that value $\rho = E / \Delta v$ is energy density of EM field.

From Planck and Einstein quantum postulates and Eq.(1) it is followed, that variations of EM signal energy in time are expressed as

$$E(t) = \sum_{\mathbf{k}\alpha j} \hbar\omega_{\mathbf{k}\alpha j}\theta(t - t_{\mathbf{k}\alpha j}), \tag{2}$$

where $\theta(\cdot)$ is Heaviside step-function, reflecting the action of birth operators.

Eq. (2) is mathematical formulation of Planck quantum postulate (this formulation escaped detection in physical literature), it has and of critical importance for the following investigations.

The differentiating of Eq. (2) gives quantum instantaneous power

$$p_q(t) = \frac{\partial E(t)}{\partial t} = \sum_{\mathbf{k}\alpha j} \hbar\omega_{\mathbf{k}\alpha j}\delta(t - t_{\mathbf{k}\alpha j}), \tag{3}$$

which is described by amplitude and pulse rate modulated sequence of $\delta-$functions. At this take place function $p_q(t)$ contains information about the polarization of photons, forming EM signal, and the directions of their motion.

Being an energetic characteristic, instantaneous power does not give exhausting representation of signal, and it is necessary to take into account the orientations of spins of photons, composing signal, for total description of EM signal. Then the law of signal forming is given by the follow vector-function

$$\mathbf{L}(t) = (p_q(t), \mathrm{spin}_{\mathbf{k}\alpha}(t)), \quad (4)$$

where $\mathrm{spin}_{\mathbf{k}\alpha}(\cdot)$ is signum function, describing the orientation of spins of photons.

From Eqs. (3) and (4) it follows that photons carry information about "sign" and "polarization" of sorts of quantum jump, causing the birth of photon with one or another orientation of spin and polarization.

The oscillations $\mathbf{L}(t)$ of sources of the type (4) induce EM radiation beam, that is described by vector-function

$$\mathbf{L}(t,\mathbf{R}) = (\sum_{\mathbf{k}\alpha j} \hbar\omega_{\mathbf{k}\alpha j}\delta[c_0(t - t_{\mathbf{k}\alpha j}) - \\ -(\mathbf{R}_{\mathbf{k}\alpha j} - \mathbf{r}^0)\cdot\mathbf{n}], \mathrm{spin}_{\mathbf{k}\alpha}(t)), \quad (5)$$

where $\mathbf{R}_{\mathbf{k}\alpha}\cdot\mathbf{n} = |\mathbf{R}_{\mathbf{k}\alpha}|$.

The argument of $\delta-$function in Eq. (5) and stochastic character of direction $\mathbf{R}_{\mathbf{k}\alpha j}$ allow to conclude that photons should be regarded as point objects, similar to electrons, which quantum electrodynamics assumes as point objects. That allows to describe EM radiation as

$$\mathbf{L}(t,\mathbf{R}) = \frac{1}{h_1h_2h_3}(\sum_{\mathbf{k}\alpha j} \hbar\omega_{\mathbf{k}\alpha j}\delta[c_0(t - t_{\mathbf{k}\alpha j}) - \\ -(q_1 - q_1^0)]\cdot\delta(q_2 - q_2^0)\delta(q_3 - q_3^0)), \quad (6)$$

where h_1, h_2, h_3 are Lame coefficients; q_1, q_2, q_3 are arbitrary orthogonal curvilinear coordinates; $q_1 - q_1^0 = (\mathbf{R}_{\mathbf{k}\alpha} - \mathbf{r}^0)\cdot\mathbf{n}$.

Space-time process $\mathbf{L}(t,\mathbf{R})$ is the sequences of corpuscular representation of EM field, where photons are carriers of signal energy. Eqs. (6) confirms that photons can be considered as the "bunches" of energy [Ref.5], or more precisely, as "bunches" of such energy, which cause the properties of EM field [Ref. 2, 3].

If at the propagation of EM signal its instantaneous power relates to unit area element Δs, then instantaneous values of Pointing vector at exact, quantum description of EM field is expressed as

$$\mathbf{S}_q(t,\mathbf{R}) = \mathbf{n}\mathbf{L}(t,\mathbf{R})/\Delta s. \quad (7)$$

CLASSICAL APPROXIMATIONS OF ELECTROMAGNETIC SIGNALS

Classical dynamics laws are applied if and only if in given volume Δv so many quantums are born briefly so that process is seemed continuous. Its discrete nature becomes so much imperceptible that it does not detected by usual experimental methods. Probability becomes certainty in process, where great number of quantum participate, that causes dynamically determinate result. For free EM field this means that value ρ is large, i.e. in given volume Δv number N_n of photons with certain middle energy $\hbar\omega_i$ is very large, and it is possible to neglect the quantification of EM field [Ref.6].

This is the condition of classical description, it defines the condition of transfer from quantum description of EM signal to its classical, approximate description. This is Bohr correspondence principle, which requires the coincidence of physical consequences of quantum theory in limit case of large quantum numbers. In addition photons, composing signal, are in coherent states, and indeterminateness relations take minimal values.

However correspondence principle does not indicate the ways of establishment of connection between quantum and classical description of physical phenomena, and the theme of this connection, i.e. joining of quantum and classical physics is a global problem, which is yet unsolved [Ref. 1].

Connection between quantum and classical description of EM field is established in the follow context. Polarization $\mathbf{e}$ can be represented in the form [Ref. 4]

$$\mathbf{e} = e_1\mathbf{e}_1 + e_2\mathbf{e}_2, \quad (8)$$

where $\mathbf{e}_1, \mathbf{e}_2$ are mutually orthogonal polarizations.

Linear polarizations $\mathbf{e}_1, \mathbf{e}_2$ are chosen, and transfer is made from the number of photons in $\mathbf{k}-\alpha$ states to EM wave with linear polarization and wave vector $\mathbf{k}_0$.

The establishment of connection between quantum and classical description of EM signals is based on Lorentz idea [Ref.7] of replacement of discrete physical object by continuous object, and, besides, on the assumption [Ref. 4] that classical EM field should be considered as time-averaged.

In this case discrete physical object is quantum instantaneous power $p_q(t)$ of linearly polarized EM radiation Eq. (3) (where summing index $\mathbf{k}\alpha j$ is replaced by j), and final results is classical function $s(t)$, which describes the oscillations of EM field strengths.

Transition to oscillation $s(t)$ is realized by two stages. The first stage is the development of Lorentz idea and is aimed to the obtaining of functional relation, which describes classical instantaneous power $p(t) = s^2(t)$ in the form of continuous one-dimensional object. That functional relation is given by convolution in distribution space [Ref.8]

$$p(t) = \left\langle \sum_j \hbar\omega_j\delta(t' - t + t_j), f(t') \right\rangle, \quad (9)$$

where $f(\cdot)$ is a suitable interpolating function.

Operation (9) is conveniently interpreted [Ref. 2, 3] in terms of theory of transmission of continuous signals by the methods of pulse modulation. As this take place, volumes $\hbar\omega_{)}$ are understood as sampling values at non-optimal sampling with a great number of excess sampling. In other words in the condition of classical description time intervals $,_{)} - ,_{)-1}$ are much less than in the case of optimal sampling, and function $(_{s}(,)$ is an ideal sequence for obtaining of continuous function in the form of classical instantaneous power $((,)$. There exist a freedom of choice of interpolating function $p(\cdot)$ fort very small non-optimal values $,_{)} - ,_{)-1}$. From the theoretical physics standpoint [Refs. 4, 7], interpolating function $p(\cdot)$ should be rectangular, i.e. $p(,) = \mathrm{rect}(, / \Delta,)$, where its duration $\Delta,$ shoul not exceed time interval, in which EM field can vary considerably. The field should be sufficiently strong, the more strength, the less averaging interval $\Delta,$. Function $p(\cdot)$ is normalized so that the increment Δi of signal energy for any time interval $(,,, + \Delta,)$ is given, with a high degree precision, by

$$\Delta i\ (,,\Delta,) = \int_{,}^{,+\Delta,} ((,')[,' =]_{n}(,,\Delta,)\hbar\omega_{n}, \quad (10)$$

where $]_{n}(,,\Delta,)$ is the number of photons with middle energy $\hbar\omega_{n}$, birthing for time interval $(,,, + \Delta,)$.

Vast number of excess samples in function $(_{s}(,)$ explains the invariance of diffractional pattern at the output plane of classical optical spectrum analyzer at the decrease of level of analyzed optical signal and the increase of exposure time. The decrease of analyzed radiation intensity means the decrease of number of these samples, but the increase of exposure time retains information about statistic properties of source because the number of photons onto spectrum analyzer output plane remains approximately the same.

The second stage consists in transfer to signal $0(,)$ by taking sign-alternating square root from the Eq. (9) of classical instantaneous power. Sign-alternating character of classical function $0(,)$ requires distinguishing of photons, corresponding to its positive and negative values. Photons, as identical particles, can differ either by space or by spin orientation. Here photons differ by the orientation of their spins, that defines sign-alternating character of EM field oscillations, which is expressed as

$$0(,) = \sqrt{((,)} \cdot \mathrm{spin}(t) = |0(,)| \cdot \mathrm{spin}(,). \quad (11)$$

The role of orientation of spins of photons was illustrated graphically in the author's papers [Ref. 2,3].

Wave character of EM signal $\mathbf{L}(,,\mathbf{R})$ is sensed at the sacrifice of large quantities of photons (i.e. particles), that takes place in the condition of classical description.

In classical electrodynamics the modulus of Pointing vector is expressed as

$$|\mathbf{S}| = |\mathbf{E} \times \mathbf{H}| = \sqrt{\varepsilon / \mu}|\mathbf{E}|^{2} = \sqrt{\mu / \varepsilon}|\mathbf{H}|^{2}, \quad (12)$$

where $\mathbf{E}, \mathbf{H}$ are electrical and magnetic vectors of EM field; $\varepsilon, \mu -$ are inductive capacity and magnetic inductive capacity.

Alternatively, $|\mathbf{S}|$ is expressed by averaged values of classical instantaneous power in the form

$$|\mathbf{S}| = \frac{((,)}{\Delta 0}, \quad (13)$$

therefore function $0(,)$ can describe the oscillation of EM field strengths.

In the case of optical frequency standards the objects of measurements are the characteristics of electrical vector oscillations, and in the case of microwave frequency standards the objects of spectral measurements are spectral functions of oscillations of current and voltage. These classical oscillations result from Ohm laws. So for current the follows relations take place:

$$\mathbf{j}(,) = \sigma\mathbf{E}(,), \text{п}(,) = \iint_{\Delta 1} \mathbf{j}(,)[\mathbf{S}. \quad (14)$$

where σ is specific conductivity; $\Delta 1$ is certain surface.

Electromagnetic signals $0(,)$ in the forms of current or voltage oscillations, and EM field strength are the objects of transformations at classical description of physical phenomena. These classical oscillations were obtained from exact, quantum description of EM field by time averaging (9) of discrete structure of signal (3). Therefore these oscillations are nonexistent objects in nature [Ref 9]. These are abstract oscillating processes the modern signal theory and information theory deal with. For this reason the construction of classical mathematical signals models, based on analytical functions, and conclusions of information theory [Ref.10], must take this condition into account. The ignoring of this condition is the reason of contradictions and difficulties of modern signal theory, which were considered in earlier author's researches (see [Refs 2, 3]).

ELECTROMAGNETIC SIGNALS IN TWO-LEVEL SYSTEMS

The law of signal forming (4) describes in general case the variations of states of certain quantum object. Electromagnetic signal formed in quantum system with two possible stationary states with energies i_{3} and $i_{]}$ is a special case. As this takes place, both spontaneous and induced oscillations of photons with circular frequency $\omega_{3]} = |i_{3} - i_{]}|/\hbar$ are possible. Electromagnetic signals of frequency standards are usually generated in such systems. The law of EM signal forming in two-level system is defined by $\delta -$ function repetition rate, polarization state, wave normal $\mathbf{n}$ and the orientation of spins of photons.

Spontaneous radiation is a result of such birth operator actions, which cause "bunches" of energy characterized

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not only by random time of appearance, but also by polarization. Furthermore, correlation between polarization of isolated “bunches” of energy is absent.

In two-level system the law of forming of linear polarized EM signal is defined by the repetition rate of $\delta -$ function and the orientation of spins of photons. In this case at the conditions of classical description averaged instantaneous power is given as

$$p(t) = \hbar\omega_{mn} \sum_j \left\langle \delta(t' - t + t_j), f(t') \right\rangle. \quad (15)$$

In the case of spontaneous radiation the moments t_j of action of birth operators in Eq. (15) are stochastic, and they correspond to the classical oscillation $s(t)$ with random amplitude and phase, as sign-alternating square root from function (15). These signals are among natural and considered as amplitude and phase modulated.

Monochromatic induced radiation is formed by action of birth operators with such periodic regularity that vector-function $\mathbf{L}(t)$ becomes periodic. This radiation is polarized, and its time oscillations are described by classical harmonic function

$$s(t) = S_0 \cos(\omega_{mn} t + \varphi), \quad (16)$$

its period is $T_{mn} = 2\pi / \omega_{mn}$, and classical instantaneous power is given by

$$p(t) = P_0 [1 + \cos 2(\omega_{mn} t + \varphi)]. \quad (17)$$

Conversely, at monochromatic radiation the sequence of $\delta -$ functions in Eq. (15) is pulse rate modulated according to the law (17) of instantaneous power variations of harmonic oscillation

At transfer to oscillation $s(t)$ in the form (16) the sign of square root from $p(t)$ in Eq. (17) is defined by signum function

$$\mathrm{spin}(t) = \mathrm{sgn} \cos(\omega_{mn} t + \varphi), \quad (18)$$

The classical value-circular frequency $\omega_{mn} = 2\pi / T_{mn}$ of harmonic oscillation (16), which according to Planck quantum postulatr, defines photon energy $E = \hbar\omega_{mn}$, is expressed by the period $T_{mn} = 2\pi / \omega_{mn}$ .Eq. (16) is idealized description of oscillations of active quantum frequency standards, its spectrums are described by the function of the form $\delta(\omega - \omega_{mn})$. In the context of classical description real signals of active quantum frequency standards are very narrow-band oscillations with random amplitude and phase, that is stipulated by stochastic nature of quantum processes and a number of physical and technical factors. As a results spectral line of frequency standards has finite width. From physical standpoint quantum and classical width of spectrum of frequency standards oscillations can be introduced. Quantum width of spectrum is defined by the scattering of frequencies of photons, and classical width- by the scattering of frequencies of photons and by the pulse rate modulation of $\delta -$ functions in Eq. (15).

CONCLUSON

Represented results of researches are composite part of physical signal theory. This theory allows to describe signals on physical level, as differentiated from modern signal theory, which is a formal mathematical discipline, an applied partition of functional analysis. Physical signal theory, constructed on the base of quantum representations, gives the most consistent and full consideration of EM signals properties, in particular of the signals of optical and microwaves quantum frequency standards.

Physical signal theory is the base of developing of unique approach to quantum objects- EM signals, in particular the signals of optical and microwaves quantum frequency standards and interacting with this signals measuring apparatus.

Therefore, fulfilled researches are contribution in the first stage of solving the fundamental problem of joining the quantum and classical physics in the context of EM field theory.

Like it is assumed [Ref.1], this contribution was done “Thanks to going beyond traditional quantum formalism and the creation of some metaphysics”. That metaphysics is in the principles of physical signal theory, represented in this paper.

REFERENCIES

[1] Khlyshko D.N. Physics Uspekhi 1998. No. 9. P. 975-1015.(in Russian)

[2] Moskaletz O.D. VUZ Izvestiya. Physics. 2001. No. 10. P. 5-12. (in Russian).

[3] Moskaletz O.D. Russian Physic Journal. 2001. No. 10. P. 3-12.

[4] Berestetsky V.B., Lifshits E.M., Pitayevsky L.P. Quantum electrodynamics. The second edition. Moskow, Nauka. 1980. (in Russian)

[5] Stepanov B.I. Introduction into modern optics. Quantum theory of light and matter interaction. Minsk, Navuka and techniques. 1990. (in Russian).

[6] Bohm D. Quantum theory. New York, Prentice-Hall, Inc. 1952.

[7] Lorentz H. A. The Theory of Electrons and Its Applications to the Phenomena of Light and Radiation Heat/– 2ed – New York: Dover, cop.1952.

[8] Moskaletz O.D. XV .All Union Conf. “Acoustoelectronics and Physical Acoustics of Solid”. Leningrad. 1991. Pt.III. P. 104-105. (in Russian).

[9] Vichman E. Quantum Physics. Berckley Physics Course. V. Moskow. Nauka. 1986. (in Russian).

[10] Zheleznov N.A. Radio engineering. 1957. No 11. P.3-13. (in Russian).

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